Visual-Inertial-Wheel Odometry with Online Calibration

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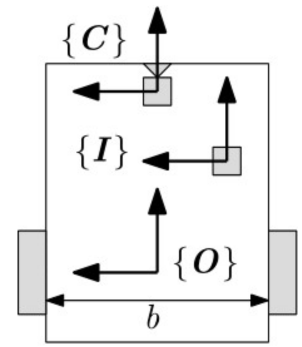
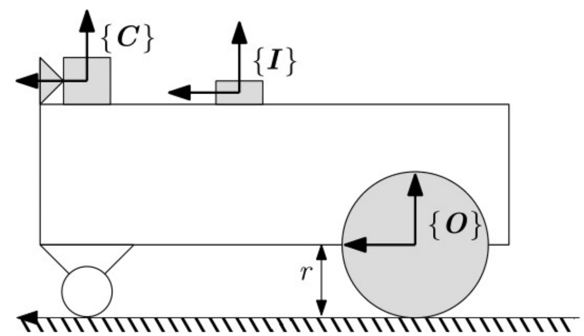
Motivation

- Efficient and accurate multi-sensor motion tracking (odometry) is essential for long-term autonomy of **ground vehicles**
- Ground vehicles typically equipped with low-cost and multi modal sensors such as wheel encoders, cameras and IMUs
- Application of Visual-Inertial odometry (VIO) is not robust for a ground vehicles due to its **constrained motions** (e.g. constant velocity, planar motion)
- Fusing wheel encoder measurement with VIO can provide the **absolute scale** to improve robustness of the odometry system
- Thorough investigation into **online calibration** of wheel encoders spatiotemporal extrinsics and intrinsics, crucial to handling varying environmental conditions, has not been fully explored





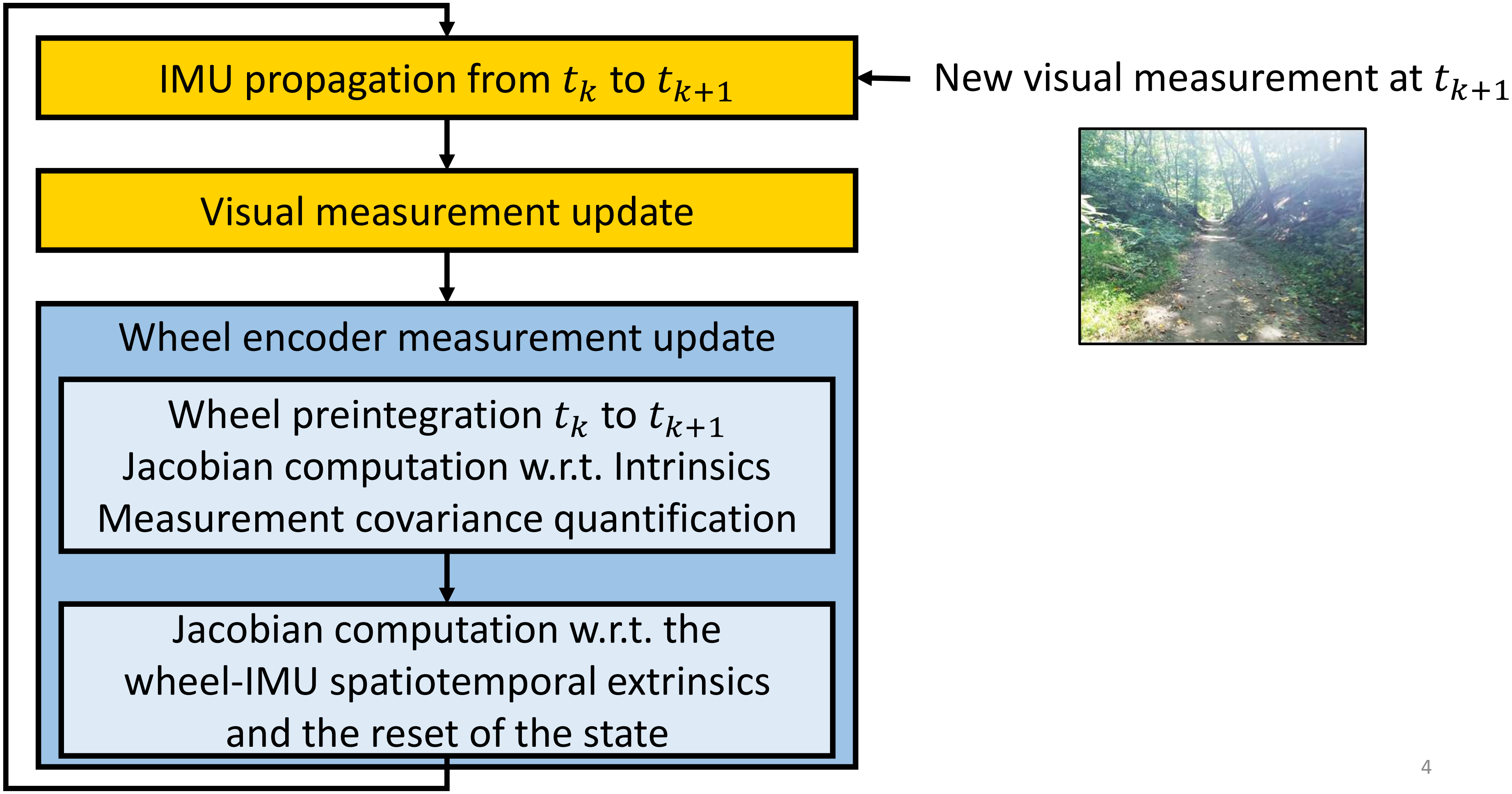
Contributions

- Tightly-coupled **Visual-Inertial Wheel Odometry (VIWO)** efficiently fusing IMU, camera, and wheel encoder measurements

- Modeling of the wheel-IMU time offset and **online calibration** of both **intrinsic and extrinsic spatiotemporal** calibration parameters

- Observability analysis to prove **4 unobservable directions** under general motions and identify **5 degenerate motions** for calibration

Overview of VIWO



VIWO State

- VIWO is based on MSCKF framework
- The state vector

$$\mathbf{x}_k = \begin{bmatrix} \mathbf{x}_{I_k} & \mathbf{x}_{C_k} & \mathbf{x}_{WE} & {}^{O}t_I & \mathbf{x}_{WI} \end{bmatrix}$$

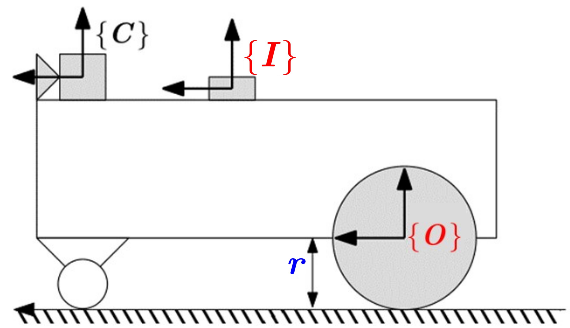
$$\mathbf{x}_{I_k} = \begin{bmatrix} {}^{I_k}_{G}\bar{q} & {}^{G}\mathbf{p}_{I_k} & {}^{G}\mathbf{v}_{I_k} & \mathbf{b}_{\omega_k} & \mathbf{b}_{a_k} \end{bmatrix}$$

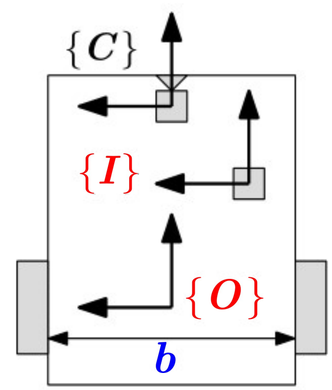
$$\mathbf{x}_{C_k} = \begin{bmatrix} {}^{I_{k-1}}_{G}\bar{q} & {}^{G}\mathbf{p}_{I_{k-1}} & \cdots & {}^{I_{k-n}}_{G}\bar{q} & {}^{G}\mathbf{p}_{I_{k-n}} \end{bmatrix}$$

$$\mathbf{x}_{WE} = \begin{bmatrix} {}^{O}_{I}\bar{q} & {}^{O}\mathbf{p}_{I} \end{bmatrix} : \text{Wheel to IMU extrinsics}$$

$${}^{O}t_I : \text{Wheel to IMU time offset}$$

$$\mathbf{x}_{WI} = \begin{bmatrix} r_l & r_r & b \end{bmatrix} : \text{Wheel intrinsics}$$





Wheel Odometry Preintegration

- Performing EKF update at the wheel encoder measurement rate (10^2- 10^3 Hz) is **too expensive**

- Integrating the wheel measurements between t_k and t_{k+1} provides the inferred measurement at lower rate:

$$\mathbf{z}_{k+1} = \mathbf{g}(\{{}^{O_i}\omega, {}^{O_i}v\}_{i=k:k+1}) = \begin{bmatrix} \int_{t_k}^{t_{k+1}} {}^{O_t}\omega dt \\ \int_{t_k}^{t_{k+1}} {}^{O_t}v \cos({}^{O_t}_{O_k}\theta) dt \\ \int_{t_k}^{t_{k+1}} {}^{O_t}v \sin({}^{O_t}_{O_k}\theta) dt \end{bmatrix}$$

  - ${}^{O}\omega$ and ${}^{O}v$ are inferred from the raw measurements by using intrinsics $\mathbf{x}_{WI}$

$$ {}^{O}\omega = (\omega_r r_r - \omega_l r_l)/b \ , \qquad {}^{O}v = (\omega_r r_r + \omega_l r_l)/2$$

  - The integrated measurement can be approximated through linearization by fixing the value of intrinsics at the current estimate $\hat{\mathbf{x}}_{WI}$:

$$\mathbf{z}_{k+1} \simeq \mathbf{g}(\{\omega_{l_i}, \omega_{r_i}\}_{i=k:k+1}, \hat{\mathbf{x}}_{WI}) + \frac{\partial \mathbf{g}}{\partial \tilde{\mathbf{x}}_{WI}} \tilde{\mathbf{x}}_{WI} + \frac{\partial \mathbf{g}}{\partial \mathbf{n}_w} \mathbf{n}_w$$

$$\mathbf{x}_{WI} = \begin{bmatrix} r_l & r_r & b \end{bmatrix} : \text{Intrinsics}$$

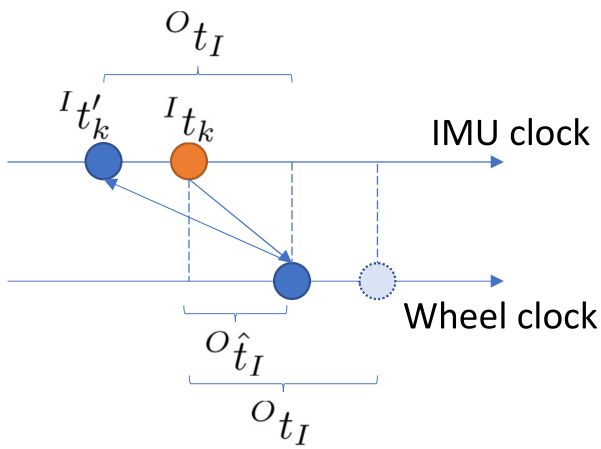
Time Offset Model

- Model an unknown constant time offset between the IMU and the Wheel:

$$^{I}t_k = {}^{O}t_k + {}^{O}t_I$$

- When integrating the wheel measurement from $^{I}t_k$ to $^{I}t_{k+1}$, the first wheel measurement is picked using the current estimate $^{O}\hat{t}_I$ which is in IMU clock:

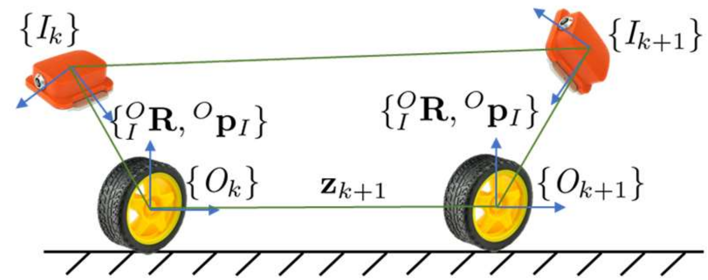
$$^{I}t'_k := {}^{I}t_k - {}^{O}\hat{t}_I + {}^{O}t_I = {}^{I}t_k + {}^{O}\tilde{t}_I$$



- We employ the following first-order approximation to account for this small time offset error:

$$^{I'_k}_{G}\mathbf{R}({}^{O}\tilde{t}_I) \approx (\mathbf{I} - \lfloor {}^{I_k}\boldsymbol{\omega}\,{}^{O}\tilde{t}_I \rfloor)\,^{I_k}_{G}\mathbf{R}$$
$$^{G}\mathbf{p}_{I'_k}({}^{O}\tilde{t}_I) \approx {}^{G}\mathbf{p}_{I_k} + {}^{G}\mathbf{v}_{I_k}\,{}^{O}\tilde{t}_I$$

Measurement Function



- The 2D measurement function expressed with 3D state:

$$\mathbf{z}_{k+1} = \mathbf{h}(\mathbf{x}_{k+1}) = \begin{bmatrix} \mathbf{e}_3^\top \mathrm{Log}\left({}^O_I\mathbf{R}\, {}^{I'_{k+1}}_{G}\mathbf{R}({}^O\tilde{t}_I)\, {}^{I'_k}_{G}\mathbf{R}({}^O\tilde{t}_I)^\top\, {}^O_I\mathbf{R}^\top\right) \\ \Lambda\left({}^O_I\mathbf{R}\, {}^{I'_k}_{G}\mathbf{R}({}^O\tilde{t}_I)\left({}^G\mathbf{p}_{I'_{k+1}}({}^O\tilde{t}_I) + {}^{I'_{k+1}}_{G}\mathbf{R}({}^O\tilde{t}_I)^\top\, {}^I\mathbf{p}_O - {}^G\mathbf{p}_{I'_k}({}^O\tilde{t}_I)\right) + {}^O\mathbf{p}_I\right) \end{bmatrix}$$

  - $\mathbf{e}_1$ and $\Lambda = [\mathbf{e}_1 \ \mathbf{e}_2]^\top$ projects 3D pose onto the 2D plane

- Linearize the measurement function and get the Jacobians w.r.t. inertial/clone states, and wheel spatiotemporal extrinsics:

$$\mathbf{z}_{k+1} \approx \mathbf{h}(\hat{\mathbf{x}}_I, \hat{\mathbf{x}}_C, \hat{\mathbf{x}}_{WE}, {}^O\hat{t}_I) + \frac{\partial \mathbf{h}}{\partial \tilde{\mathbf{x}}_I}\partial\tilde{\mathbf{x}}_I + \frac{\partial \mathbf{h}}{\partial \tilde{\mathbf{x}}_C}\partial\tilde{\mathbf{x}}_C + \frac{\partial \mathbf{h}}{\partial \tilde{\mathbf{x}}_{WE}}\partial\tilde{\mathbf{x}}_{WE} + \frac{\partial \mathbf{h}}{\partial {}^O\tilde{t}_I}\partial {}^O\tilde{t}_I$$

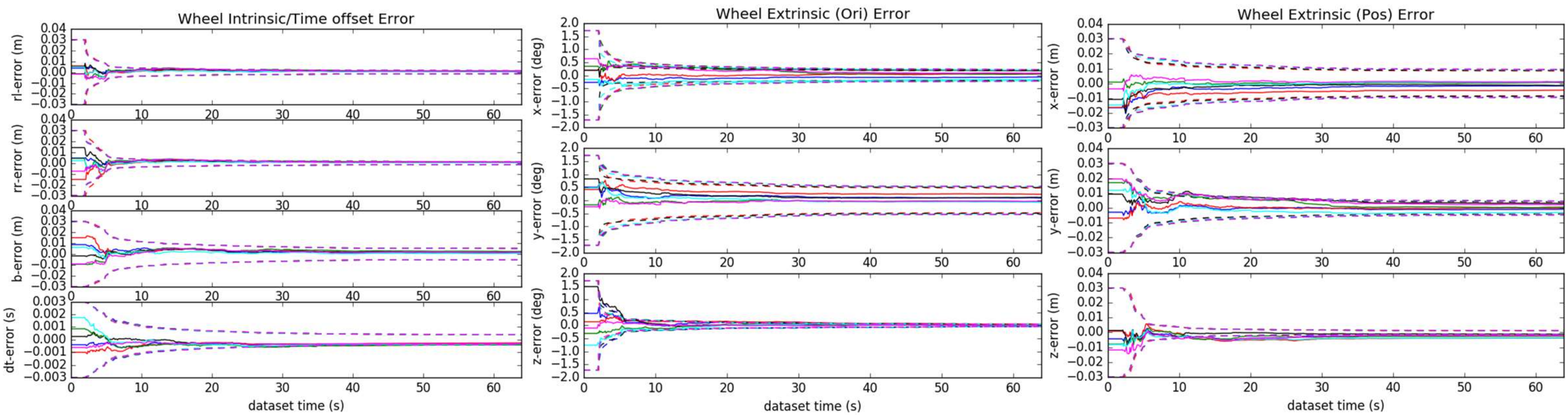
- Perform update using computed Jacobian matrices

$$\tilde{\mathbf{z}}_{k+1} := \mathbf{g}(\underset{i=k:k+1}{\{\omega_{l_i}, \omega_{r_i}\}}, \hat{\mathbf{x}}_{WI}) - \mathbf{h}(\hat{\mathbf{x}}_I, \hat{\mathbf{x}}_C, \hat{\mathbf{x}}_{WE}, {}^O\hat{t}_I) \approx \underbrace{\begin{bmatrix} \frac{\partial \mathbf{h}}{\partial \tilde{\mathbf{x}}_I} & \frac{\partial \mathbf{h}}{\partial \tilde{\mathbf{x}}_C} & \frac{\partial \mathbf{h}}{\partial \tilde{\mathbf{x}}_{WE}} & -\frac{\partial \mathbf{g}}{\partial \tilde{\mathbf{x}}_{WI}} & \frac{\partial \mathbf{h}}{\partial {}^O\tilde{t}_I} \end{bmatrix}}_{\mathbf{H}_{k+1}} \tilde{\mathbf{x}}_{k+1} - \frac{\partial \mathbf{g}}{\partial \mathbf{n}_\omega}\mathbf{n}_\omega$$

Log() is the SO(3) matrix logarithm function

Simulation Results: Calibration

- **All calibration converges** under general 3D motion
- **Consistent calibration:** the errors(solid) remain between 3σ bounds(dotted)

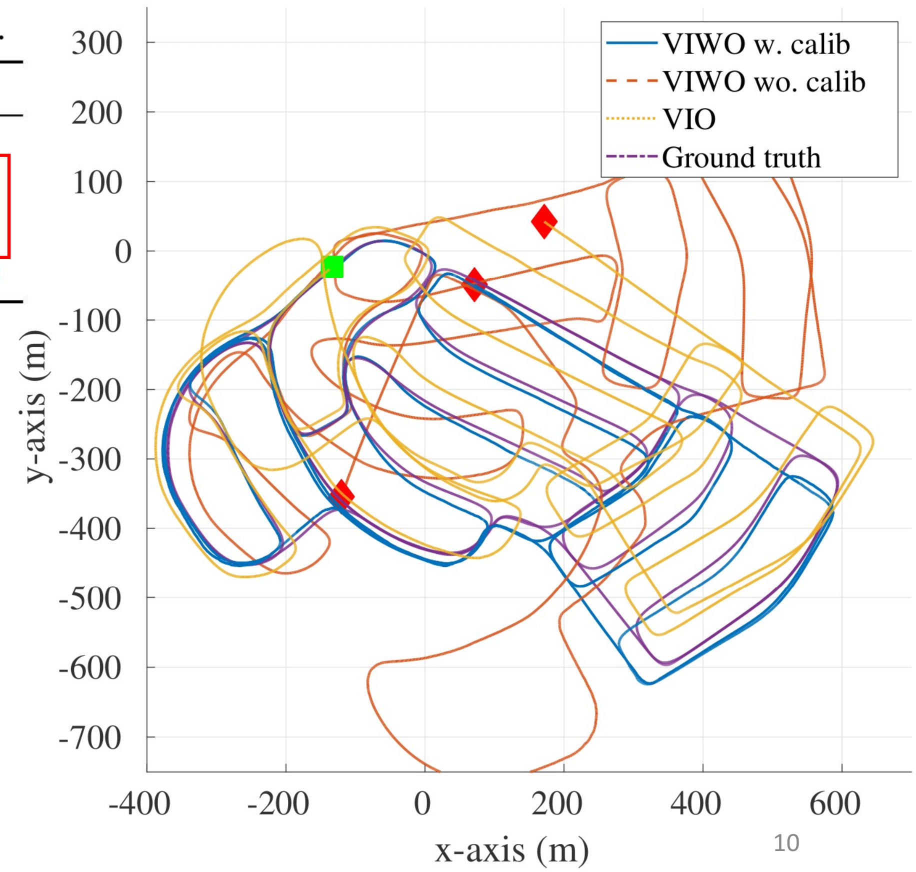


Simulation Results: Localization

TABLE II: Relaive pose error (RPE) of each algorithm (degree/meter).

	50m	100m	200m	NEES
VIO	0.362 / 1.252	0.494 / 2.245	0.657 / 3.930	3.921 / 3.895
true & w. cal.	0.277 / 0.550	0.365 / 0.908	0.479 / 1.573	1.952 / 2.020
true & wo. cal.	0.259 / 0.384	0.340 / 0.622	0.443 / 1.125	1.698 / 1.473
bad & w. cal.	0.276 / 0.543	0.365 / 0.888	0.486 / 1.526	1.943 / 1.826
bad & wo. cal.	0.572 / 0.510	1.104 / 1.142	2.239 / 3.367	59.678 / 183.538

- 8.9km simulated trajectory
- The calibration allows the estimator to be **consistent** and **accurate** near to the performance given the true values



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Degenerate Motion Analysis

- The system has **4 unobservable directions** global position and yaw
- Calibration parameters are fully observable given general motions.
- Identified **5 degenerate motions** causing the calibration parameters to become unobservable through observability analysis

Motion	**Unobservable**
Pure translation	${}^{O}\mathbf{p}_I$, b
1-axis rotation	${}^{O}\mathbf{p}_I$ along the axis
Constant angular and linear velocity	${}^{O}t_I$
No left/right wheel velocity	r_l / r_r
No motion	${}^{O}_{I}\mathbf{R}$, ${}^{O}\mathbf{p}_I$, r_l, r_r, b

Online Calibration during Planar Motion
(Degenerate Motion Analysis)

Conclusion

- Proposed **tightly coupled** visual-inertial-wheel odometry (VIWO)
  - **Efficient integration** of wheel encoder measurements for 3D motion tracking with proper uncertainty quantification
  - Rigorous online sensor calibration of **spatiotemporal extrinsics** of wheel-IMU and wheel encoder's **intrinsics**
- Proved the state contains **4 unobservable directions** under general motions and identified **5 degenerate motions** for calibration

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For full video